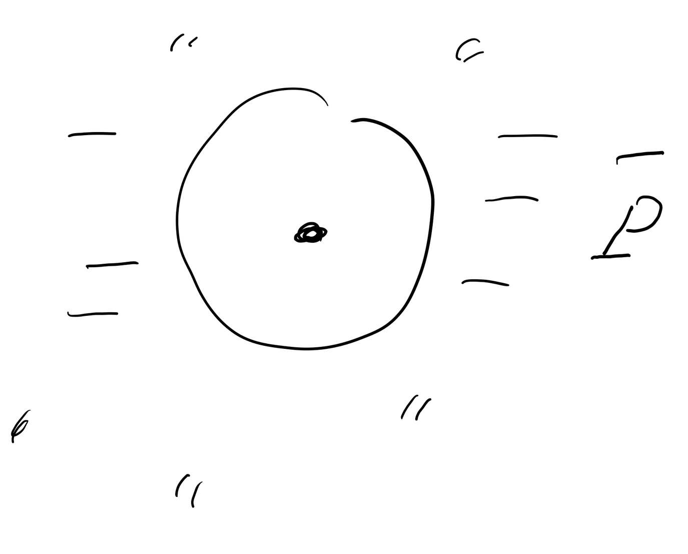
At this point we need to generalize the case and forget that we have low density medium.

Let's now consider inner field that may come from neighboring dipoles.

Ludwig Lorenz (Denmark, 1869)　　Hendrik Lorentz (Netherlands, 1878)

Lorenz have shown the following. If we take homogeneous dielectric, then the field in some empty space inside it will be:



$$\bar{E}' = \bar{E} + \underbrace{\frac{1}{3\varepsilon_0}\bar{P}}_{\text{inner field}}$$

This is the field of surrounding dipoles.

Now we assume the case of transparent medium ($\varkappa = 0$)

$$\ddot{\bar{r}} + \omega_0^2 \bar{r} = -\frac{e}{m}\bar{E}' = -\frac{e}{m}\left(\bar{E} + \frac{1}{3\varepsilon_0}\bar{P}\right)$$

Second Newton's law

Let's multiply both parts by $-Ne$.

$$-Ne\ddot{\bar{r}} + (-Ne\omega_0^2\bar{r}) - \frac{Ne^2}{3m\varepsilon_0}\bar{P} = \frac{Ne^2}{m}\bar{E}$$

We remember, that $\bar{P} = -eN\bar{r}$

Thus

$$\ddot{\bar{P}} + \left(\omega_0^2 - \frac{e^2N}{3m\varepsilon_0}\right)\bar{P} = \frac{Ne^2}{m}\bar{E}$$

Again, this is an equation of forced oscillations. But now it is oscillation of polarization of medium dipoles.

The solution:

$$P = \frac{\frac{Ne^2}{m}\bar{E}}{\left(\omega_0 - \frac{e^2N}{3m\varepsilon_0}\right) - \omega^2}$$

But $P = \varepsilon_0 \chi \bar{E} \Rightarrow \chi$

Since we know χ, we can determine ε.

$$n^2 = \varepsilon = 1 + \chi = 1 + \frac{Ne^2}{m\varepsilon_0} \cdot \frac{1}{\omega_0^2 - \omega^2 - \frac{Ne^2}{3m\varepsilon_0}}$$

Let's build the following combination

$$\boxed{\frac{n^2 - 1}{n^2 + 2} = \frac{Ne^2}{3m\varepsilon_0} \cdot \frac{1}{\omega_0^2 - \omega^2}}$$

Lorentz-Lorenz equation

Homework derive

For the given frequency we can derive:

$$\frac{n^2-1}{n^2+2} \cdot \frac{1}{N} = \text{const}$$

According to this formula, by measuring n we can determine concentration N.

$$\underbrace{\frac{n^2-1}{n^2+2} \cdot \frac{1}{\rho}}_{\text{specific refraction}} = \text{const} \qquad \rho \text{ - density}$$

From this equation it is clear, that refractive index and density of medium are connected.

Why stars twinkle, but planets don't

Let's specifically talk about gases. For them $n \simeq 1$

In this case

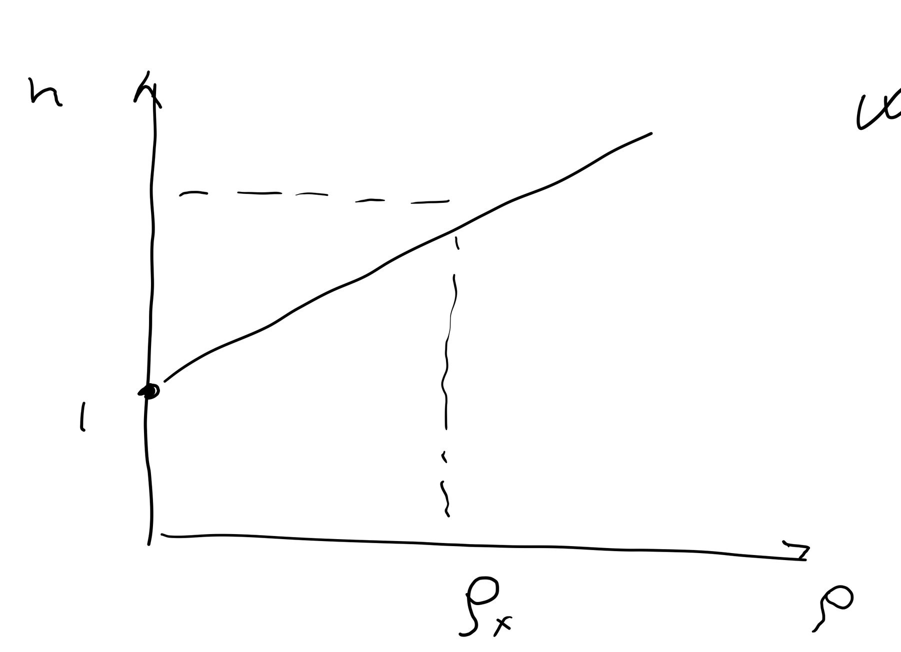
$$n^2 - 1 = (n-1)(n+1) \simeq 2(n-1)$$

$$n^2 + 2 \simeq 3$$

It means that specific refraction:

$$n - 1 = \rho \cdot \text{const}$$

$$n = 1 + \text{const}\,\rho$$



We can determine density of gas.